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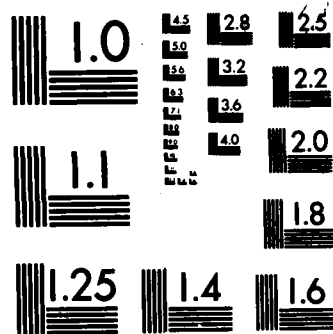
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UNSTEADY COMPRESSIBLE LAMINAR
BOUNDARY-LAYER FLOW WITHIN
AN EXPANSION WAVE

Lang-Mann Chang

July 1983

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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20. ABSTRACT (Continued):

distribution in the boundary layer. It is shown that both velocity and temperature boundary layers grow rapidly behind the expansion wavefront but attenuate further downstream. If the wall temperature is held at the undisturbed temperature, the heat transfer occurs from the wall to the expanding gas. The rate of heat transfer increases from the wavefront to approximately $\xi = 0.35$ behind the wave where maximum heat transfer is observed, ξ being a dimensionless distance from the expansion wavefront. The isothermal wall temperature is found to be higher than the adiabatic wall temperature by a factor of $(1 + 0.335\xi)$.

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I. INTRODUCTION

In the present report we study the velocity and temperature boundary-layer development within a centered expansion wave moving into a stationary fluid. A centered expansion wave can be generated either in a shock tube or a tube wind tunnel such as designed by Ludwig.¹ It is assumed that the fluid has initially a uniform temperature T_0 but is separated by a diaphragm in two regions, with different pressures. If the diaphragm is suddenly removed, an expansion wave propagates into the stationary high pressure region. In the expansion region, the temperature of the fluid decreases due to the drop in the pressure. Therefore, if the wall is kept at the initial temperature T_0 , there will be a net heat transfer from the wall to the fluid. This heat transfer and the boundary-layer development in the expansion region is the subject of the present analysis.

As early as 1859, Riemann² analyzed the propagation of a plane disturbance of finite amplitude in an unsteady one-dimensional isentropic flow. Subsequently, many other authors studied the shock tube problem. Huber et al.,³ for example, provided analytical expressions for the velocity, pressure, and temperature of the inviscid flow. However, in order to calculate the heat transfer characteristics, the viscous boundary-layer development on the inner wall must be computed. Mirels⁴ analyzed the boundary-layer flow behind the expansion wave by assuming that the expansion wave is a line of discontinuity separating the undisturbed high pressure region from the rarefied gas region. Cohen⁵ used a coordinate expansion method to solve the boundary-layer flow within the centered expansion wave with a finite width. Cohen's treatment, which is an improvement over Mirels' solution, is based on the assumption that the wall is isothermal or, equivalently, that the thermal conductivity of the

¹H. Ludwig, "Tube Wind Tunnel: A Special Type of Blowdown Tunnel," AGARD Report 143, July 1957.

²B. Riemann, "Ueber die Fortpflanzung ebener Luftwellen von endlicher Schwingungsweite," Abh. der Königlichen Gesellschaft der Wissenschaften, Vol. 8, pp. 43-65, 1858-1859.

³P.W. Huber, C.E. Fitton, and F. Delpino, "Experimental Investigation of Moving Pressure Disturbance and Shock Waves and Correlation with One-Dimensional Unsteady-Flow Theory," NACA TN 1903, 1949.

⁴H. Mirels, "Boundary Layer Behind Shock or Thin Expansion Wave Moving into Stationary Fluid," NACA TN 3712, 1956.

⁵N.B. Cohen, "A Power-Series Solution for the Unsteady Laminar Boundary-Layer Flow in an Expansion Wave of Finite Width Moving Through a Gas Initially at Rest," NACA TN 3943, 1957.

wall is infinite. Hall⁶ later generalized Cohen's solution by allowing the wall to have a finite thermal conductivity, and gave solutions of heat transfer for some representative values of thermal conductivity. Hall's solution in the case of adiabatic wall predicts a temperature in the boundary layer that is equal to the temperature in the freestream.

The present study provides solutions for the boundary-layer flow corresponding to two wall thermal conditions, namely, an adiabatic condition and an isothermal condition at undisturbed fluid temperature. The solutions are obtained by applying Howarth's transformation and a similarity transformation via one-parameter groups which reduces the number of independent variables from three to two and then a series expansion. The first three terms of the series are used to obtain a solution of second-order accuracy.⁷⁻⁹

II. PROBLEM FORMULATION

The boundary-layer flow in the region of the expansion wave as shown in Figure 1 is considered to be two-dimensional, unsteady, compressible, and laminar. The fluid is ideal gas. External to the boundary layer, the flow is assumed to be unsteady, one-dimensional and inviscid. The freestream flow is assumed to be known and it is used as the matching condition for the boundary-layer solution.

⁶J.G. Hall, "Laminar Boundary Layer Developed Within Unsteady Expansion and Compression Waves," AIAA Journal, Vol. 10, p. 499, April 1972.

⁷L. Howarth, "Concerning the Effect of Compressibility of Laminar Boundary Layers and Their Separation," Proceedings of the Royal Society, Series A, Vol. 194, p. 16, 1948.

⁸A.J.A. Morgan, "The Reduction by One of the Number of Independent Variables in Some System of Partial Differential Equations," Q.J.M., p. 250, 1952.

⁹A.G. Hansen, Similarity Analyses of Boundary Value Problems in Engineering, Prentice Hall, Englewood Cliffs, NJ, 1964.

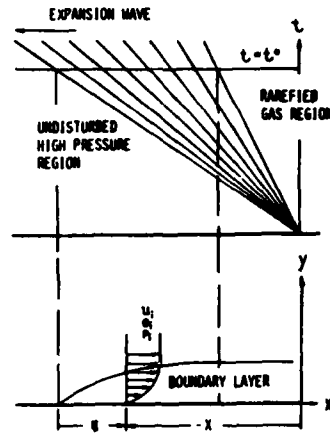


Figure 1. Shock Tube Flow at $t = t^*$

We assume that the undisturbed region is kept at a temperature T_0 . Then the wavefront of the expansion wave propagates at the sound speed of $U_r = (\gamma RT_0)^{1/2}$. The velocity, temperature, and pressure in the expansion region outside the boundary layer are given by Huber et al.³ in terms of a similarity variable ξ as follows:

Velocity

$$u_1(t, x) = \frac{2}{\gamma+1} \xi = F(\xi) \quad (1)$$

Temperature

$$\theta_1(t, x) = \frac{1}{\gamma} \left(1 - \frac{\gamma-1}{\gamma+1} \xi \right)^2 = G(\xi) \quad (2)$$

Pressure

$$p_1(t, x) = \left(1 - \frac{\gamma-1}{\gamma+1} \xi \right)^{2\gamma/(\gamma-1)} = H(\xi) \quad (3)$$

where the similarity variable ξ , derived from the similarity transformation via one-parameter groups is

$$\xi = 1 + (x/t) \quad , \quad \begin{aligned} x &= \bar{x}/L \\ t &= \bar{t}L/U_r \end{aligned} \quad (4)$$

and L is a reference length. As shown in Figure 1, $\xi=0$ denotes the expansion wavefront and $\xi=1$ is located at the origin of the wave. $F(\xi)$, $G(\xi)$, and $H(\xi)$ are, respectively, called the velocity function, temperature function, and pressure function of the inviscid flow. The velocity u_1 is the inviscid velocity normalized by U_r , the temperature θ_1 by the reference temperature $T_r = U_r^2/R$, and the pressure p_1 by $\rho_0 U_r^2$. ρ_0 is the density of the gas at rest ahead of the expansion wave.

In the boundary layer we assume that the thermal conductivity k and specific heat C_p are constants and that the viscosity $\bar{\mu}$ is proportional to the temperature. The viscosity $\bar{\mu}$ can be made dimensionless as

$$\mu = \bar{\mu}/\mu_r = T/T_r \quad (5)$$

where the reference viscosity μ_r is evaluated at the reference temperature T_r . We also define the following dimensionless variables and parameters:

$$y' = Re^{1/2} Y/L, \quad u = U/U_r, \\ v = Re^{1/2} V/U_r, \quad p = P/(\rho_0 U_r^2), \quad \theta = T/T_r, \quad \rho = \bar{\rho}/\rho_0, \quad (6)$$

$$Re = U_r L \rho_0 / \mu_r, \quad Pr = C_p \mu_r / k, \quad T_r = U_r^2 / R.$$

The governing equations for the two-dimensional, unsteady, compressible boundary-layer flow are, in terms of these variables,

Continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y'} (\rho v) = 0 \quad (7)$$

Momentum

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y'} \right) = - \frac{dp}{dx} + \frac{\partial}{\partial y'} \left(\mu \frac{\partial u}{\partial y'} \right) \quad (8)$$

Energy

$$\rho \left(\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) = \frac{1}{Pr} \frac{\partial}{\partial y} \left(u \frac{\partial \theta}{\partial y} \right) + \frac{\gamma-1}{\gamma} \left[\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + \theta \left(\frac{\partial u}{\partial y} \right)^2 \right] \quad (9)$$

State

$$p = \rho \theta \quad (10)$$

The pressure p in Eq. (8) is the same as the p_1 in Eq. (3). After introducing Howarth's transformation⁷

$$y = \int_0^{y'} \rho dy' \quad (11)$$

a streamfunction ψ , which satisfies the continuity equation, can be defined as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{1}{\rho} \left(\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} \right) \quad (12)$$

Substituting Eqs. (11) and (12) into Eqs. (8) and (9), we have

$$\frac{\partial^2 \psi}{\partial t \partial y} + \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = -\frac{\theta}{p} \frac{dp}{dx} + p \frac{\partial^3 \psi}{\partial y^3} \quad (13)$$

and

$$\frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} = \frac{p}{Pr} \frac{\partial^2 \theta}{\partial y^2} + \frac{\gamma-1}{\gamma} \left(\frac{\theta}{p} \right) \cdot \left[\frac{\partial p}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial p}{\partial x} + \frac{p^2}{\theta} \left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 \right] \quad (14)$$

In addition, we have the following boundary conditions for ψ and θ :

At the wall, $y = 0$

$$u = \frac{\partial \psi}{\partial y} = 0 \quad (\text{no slip condition}) \quad (15)$$

$$v = -\frac{1}{\rho} \frac{\partial \psi}{\partial x} = 0 \quad (\text{impermeable condition})$$

$$\theta = \theta_w = \frac{1}{\gamma} \quad (\text{isothermal wall at } T_o) \quad (16)$$

or

$$\frac{\partial \theta}{\partial y} = 0 \quad (\text{adiabatic wall}) \quad (17)$$

At the outer edge of the boundary layer, $y \rightarrow \infty$

$$u = \frac{\partial \psi}{\partial y} = u_1(t, x) \quad (18)$$

$$\theta = \theta_1(t, x) \quad (19)$$

III. METHOD OF SOLUTION

We obtain a solution of Eqs. (13) and (14) through a similarity transformation via one-parameter groups and a subsequent series expansion of the functions ψ and θ . The similarity transformation reduces the three independent variables (t, x, y) to two similarity variables (ξ, η) which are

$$\xi = 1 + (x/t) \quad \text{and} \quad \eta = y/t^{1/2} \quad (20)$$

A detailed derivation of these two similarity variables is given by Chang.¹⁰ The similarity transformation also transforms the dependent variables, ψ and θ in Eqs. (13) and (14) into

¹⁰ L.M. Chang, "Unsteady Compressible Boundary Layer Flows Within a Centered Expansion Wave and Behind a Strong Plane Blast Wave," Ph.D. Dissertation, The University of Iowa, December 1972.

$$\psi(t, x, y) = t^{1/2} f(\xi, \eta) \quad (21)$$

$$\theta(t, x, y) = g(\xi, \eta) \quad (22)$$

Substitution of Eqs. (3) and (20-22) into Eqs. (13) and (14) results in

$$(1-\xi)f_{\xi\eta} - \frac{1}{2}\eta f_{\eta\eta} + f_{\eta}f_{\xi\eta} - f_{\xi}f_{\eta\eta} = -g\frac{H'}{H} + Hf_{\eta\eta\eta} \quad (23)$$

and

$$(1-\xi)g_{\xi} - \frac{1}{2}\eta g_{\eta} + f_{\eta}g_{\xi} - f_{\xi}g_{\eta} = \frac{1}{Pr}Hg_{\eta\eta} + \frac{\gamma-1}{\gamma}\frac{g}{H} \left[(1-\xi)H' + f_{\eta}H' + \frac{H^2}{g}f_{\eta\eta}^2 \right] \quad (24)$$

where H' is the derivative of $H(\xi)$ with respect to ξ . A repeated application of the transformation technique shows that no new similarity variables that combine the variables ξ and η exist. However, we can define a locally similar variable as

$$z = \eta/\xi^{1/2} \quad (25)$$

Using the new variable z , a fast-converging series solution to the problem may be constructed as follows

$$f = \xi^{1/2}F(\xi) = \sum_{n=0}^{\infty} \xi^n f_n(z) \quad (26)$$

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial f}{\partial \eta} = F(\xi) \sum_{n=0}^{\infty} \xi^n f'_n(z) \quad (27)$$

$$\theta = g(\xi, \eta) = \frac{1}{\gamma} + \left[G(\xi) - \frac{1}{\gamma} \right] \sum_{n=0}^{\infty} \xi^n g_n(z) \quad (28)$$

Substituting the above expressions and their derivatives into Eqs. (23) and (24), we have, after grouping the terms with the same power of ξ , the following system of ordinary differential equations.

Zero-order equations ($n=0$)

$$f_0''' + (1/2)zf_0'' - f_0' = -1 \quad (29)$$

$$(1/Pr)g_0'' + (1/2)zg_0' - g_0 = -1 \quad (30)$$

First-order equations ($n=1$)

$$f_1''' + \frac{1}{2}zf_1'' - 2f_1' = \frac{2\gamma}{\gamma+1}f_0'' - \frac{3}{\gamma+1}f_0''f_0 - f_0' \quad (31)$$

$$+ \frac{2}{\gamma+1}(f_0')^2 + 2\frac{\gamma-1}{\gamma+1}g_0 - \frac{\gamma-1}{\gamma+1}$$

$$\frac{1}{Pr}g_1'' + \frac{1}{2}zg_1' - 2g_1 = \frac{1}{Pr}\frac{5\gamma-1}{2(\gamma+1)}g_0'' + \left(\frac{z}{4}\frac{\gamma-1}{\gamma+1}\right)g_0' - \left(\frac{2}{\gamma+1}\right)g_0 - \left(\frac{3}{\gamma+1}\right)g_0'f_0 + \left(\frac{2}{\gamma+1}\right)f_0'^2 \quad (32)$$

$$- \left(\frac{2}{\gamma+1}\right)f_0' + \left(\frac{2}{\gamma+1}\right)f_0'g_0 + \frac{2}{\gamma+1}$$

Second-order equations ($n=2$)

$$f_2''' + (1/2)zf_2'' - 3f_2' = F^*(z) \quad (33)$$

$$(1/Pr)g_2'' + (1/2)zg_2' - 3g_2 = G^*(z) \quad (34)$$

The expressions $F^*(z)$ and $G^*(z)$ are given in the Appendix. The boundary conditions, Eqs. (15) - (19), become:

at the wall, $z = 0$, for $n > 0$

$$f_n(0) = 0, \quad f_n'(0) = 0$$

$$g_n(0) = 0 \quad \text{for isothermal wall} \quad (35)$$

or

$$g'_n(0) = 0 \quad \text{for adiabatic wall}$$

at the outer edge of the boundary layer, $z \rightarrow \infty$

$$f'_0(\infty) = 1, \quad f'_n(\infty) = 0 \quad \text{for } n \geq 1$$

$$g_0(\infty) = 1, \quad g_n(\infty) = 0 \quad \text{for } n \geq 1 \quad (36)$$

$$g'_n(\infty) = 0 \quad \text{for } n \geq 0$$

IV. NUMERICAL RESULTS AND DISCUSSION

The preceding three sets of ordinary differential equations were solved numerically.¹⁰ With $Pr = 0.72$ and $\gamma = 1.4$ the numerical results of the velocity functions f'_0 , f'_1 , and f'_2 , and the temperature functions g_0 , g_1 , and g_2 for both isothermal and adiabatic walls are tabulated in Tables 1 and 2 and plotted in Figures 2 and 3. Since there is no temperature term involved in the zero-order momentum equation, Eq. (29), the function f'_0 is independent of the temperature condition at the wall. Therefore, as shown in Figure 2, the velocity function f'_0 is the same for the two wall conditions. However, the function f'_1 and f'_2 are certainly affected by the wall temperature. For the case of isothermal wall the present results are identical to Cohen's solutions.¹⁰

Once the zero-order, first-order, and second-order functions are computed, the solutions to the problem in a three-term series can be readily constructed.

TABLE 1. VELOCITY FUNCTIONS

z	Isothermal wall			Adiabatic wall		
	f_0'	f_1'	f_2'	f_0'	f_1'	f_2'
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.2	0.20643	0.13364	0.06425	0.20643	0.11252	0.05276
0.4	0.37735	0.22220	0.09358	0.37735	0.18962	0.08014
0.6	0.51717	0.27376	0.09495	0.51717	0.23642	0.08529
0.8	0.63011	0.29602	0.07631	0.63011	0.25834	0.07322
1.0	0.72016	0.29611	0.04555	0.72016	0.26084	0.04973
1.2	0.79100	0.28046	0.00973	0.79100	0.24907	0.02052
1.4	0.84596	0.25460	-0.02540	0.84596	0.22773	-0.00945
1.6	0.88801	0.22310	-0.05576	0.83800	0.20081	-0.03638
1.8	0.91971	0.18957	-0.07788	0.91971	0.17156	-0.05778
2.0	0.94325	0.15666	-0.09373	0.94325	0.14245	-0.07243
2.4	0.97288	0.09917	-0.10045	0.97288	0.09087	-0.08185
2.8	0.98783	0.05719	-0.08558	0.98783	0.05271	-0.07162
3.0	0.99205	0.04202	-0.07417	0.99205	0.03882	-0.06261
3.4	0.99680	0.02128	-0.05023	0.99680	0.01975	-0.04287
3.8	0.99848	0.00989	-0.03016	0.99884	0.00925	-0.02585
4.0	0.99935	0.00653	-0.02246	0.99935	0.00615	-0.01924
5.0	1.00010	0.00047	-0.00389	1.00010	0.00062	-0.00317
6.0	1.00020	0.00018	-0.00696			
7.0	1.00020	-0.00064	-0.00110			
8.0	1.00020	-0.00099	-0.00125			

TABLE 2. TEMPERATURE FUNCTIONS

z	Isothermal wall			Adiabatic wall		
	θ_0	θ_1	θ_2	θ_0	θ_1	θ_2
0	0.00000	0.00000	0.00000	1.00000	-0.18805	-0.12156
0.2	0.17756	0.03701	-0.02294	1.00000	-0.17984	-0.10638
0.4	0.32907	0.07981	-0.01369	1.00000	-0.16077	-0.07399
0.6	0.45723	0.11981	0.00384	1.00000	-0.13707	-0.03825
0.8	0.56468	0.15207	0.01733	1.00000	-0.11271	-0.00685
1.0	0.65394	0.17434	0.02195	1.00000	-0.09000	0.01685
1.2	0.72739	0.18629	0.01737	1.00000	-0.07014	0.03229
1.4	0.78705	0.18880	0.00563	1.00000	-0.05352	0.04042
1.6	0.83554	0.18346	-0.01035	1.00000	-0.04009	0.04285
1.8	0.87411	0.17213	-0.02768	1.00000	-0.02954	0.04134
2.0	0.90460	0.15673	-0.04395	1.00000	-0.02144	0.03744
2.4	0.94689	0.12032	-0.06726	1.00000	-0.01084	0.02695
2.8	0.97167	0.08461	-0.07489	1.00000	-0.00522	0.01717
3.0	0.97969	0.06891	-0.07341	1.00000	-0.00356	0.01323
3.4	0.98994	0.04334	-0.06322	1.00000	-0.00159	0.00742
3.8	0.99532	0.02553	-0.04868	1.00000	-0.00068	0.00389
4.0	0.99691	0.01916	-0.04068	1.00000	-0.00436	0.00275
5.0	0.99996	0.00383	-0.01248	1.00000	-0.00002	0.00041
6.0	1.00050	0.00081	-0.00270			
7.0	1.00060	0.00033	-0.00101			
8.0	1.00080	0.00007	-0.00100			

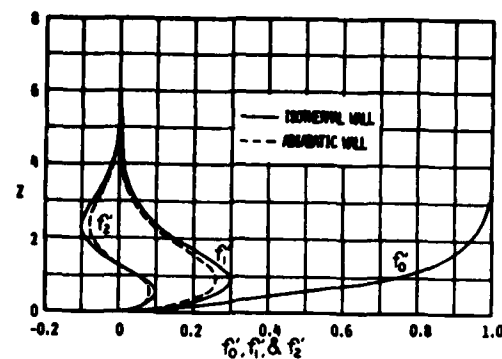


Figure 2. Profiles of Velocity Functions

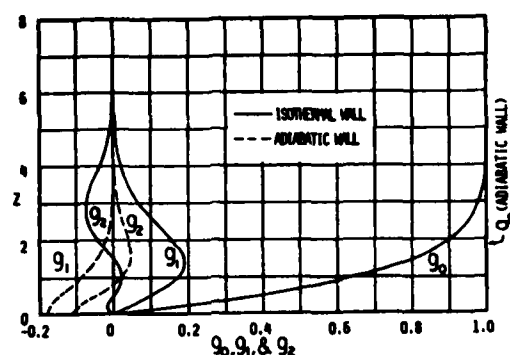


Figure 3. Profiles of Temperature Functions

A. Velocity Profiles

From Eq. (27), the velocity in the boundary layer is simply

$$u = F(f'_0 + \xi f'_1 + \xi^2 f'_2) \quad (37)$$

or

$$u/u_1 = f'_0 + \xi f'_1 + \xi^2 f'_2 \quad (38)$$

for $F = u_1$ given in Eq. (1). Figures 4 and 5 show u/u_1 vs z , respectively, for isothermal and adiabatic walls. One observes that at a given z , u/u_1 increases

with ξ . This is because, as shown in Eq. (1), the inviscid velocity u_1 relative to the wavefront appears to accelerate in the positive ξ direction. In other words, the fluid is expanding from the high pressure to the lower pressure region and the further the downstream distance is from the wavefront the higher the fluid velocity becomes. As a result, even though the viscosity diffusion in the boundary layer tends to increase the boundary growth the acceleration may suppress some growth of the boundary layer. If the boundary-layer thickness is taken to be approximately $z_\delta = 4$ from Figures 4 and 5, Eqs. (25) and (20) give

$$z_\delta = y_\delta / \sqrt{x+t} = 4 \quad (39)$$

Thus, at a given instance, the boundary-layer thickness y_δ behind the wavefront may grow like $4\sqrt{x+t}$. The larger the time, the slower the growth of the boundary-layer thickness will be with respect to x . In other words, the boundary layer grows rapidly immediately behind the expansion wavefront and flattens out further downstream from the wavefront.

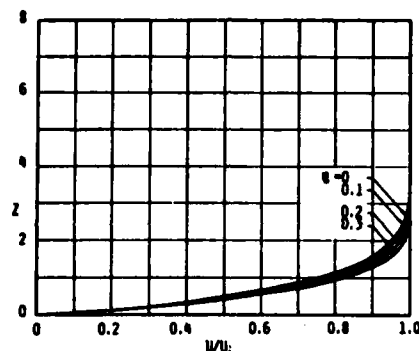


Figure 4. Velocity Profiles for an Isothermal Wall at $\xi = 0, 0.1, 0.2$, and 0.3

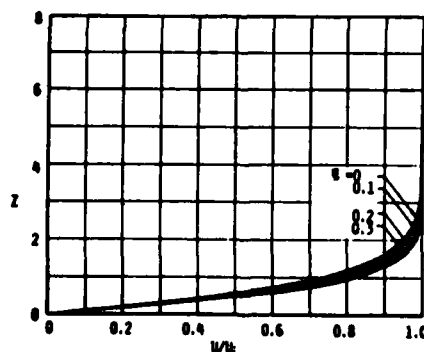


Figure 5. Velocity Profiles for an Adiabatic Wall at $\xi = 0, 0.1, 0.2$, and 0.3

A careful comparison between the profiles shown in Figures 4 and 5 reveals that at a given z the velocity is slightly higher for the isothermal wall due to heat addition from the wall to the fluid.

B. Temperature Profiles

The temperature in the boundary layer is

$$\theta = \frac{1}{Y} + \left(G - \frac{1}{Y}\right) (g_0 + \xi g_1 + \xi^2 g_2)$$

or

$$\frac{\theta}{\theta_1} = \left[\frac{1}{Y} + \left(G - \frac{1}{Y}\right) (g_0 + \xi g_1 + \xi^2 g_2) \right] / G \quad (40)$$

The solutions are plotted in Figure 6. The temperature in the boundary layer is seen to be higher than that in the freestream and the value of θ/θ_1 increases with ξ as a result of an increasing viscous heating in the boundary layer. Figure 6 shows that at $\xi = 0.3$ the wall temperature is higher than the local inviscid temperature by approximately 10 percent.

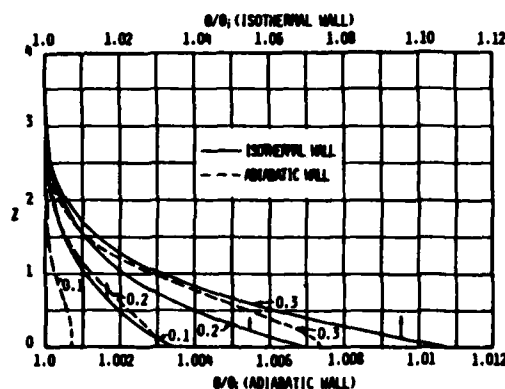


Figure 6. Temperature Profiles at $\xi = 0, 0.1, 0.2$, and 0.3

Figure 6 also shows that because of heat addition to the fluid from the wall, the surface temperature for the isothermal wall is higher than that for the adiabatic wall by 3.34 percent at $\xi = 0.1$, 6.6 percent at $\xi = 0.2$, and 10.9 percent at $\xi = 0.3$; giving the temperature ratio T_w (isothermal) to T_w (adiabatic) of approximately $(1 + 0.335\xi)$. Hall considered the case with a finite wall thermal conductivity.⁶ He found that the temperature in the

boundary layer on an adiabatic wall is the same as the temperature in the inviscid freestream. This result is equivalent to the zero-order solution (i.e., $\xi = 0$) of Eq. (40).

C. Heat Transfer

For an isothermal wall the local heat transfer from the wall to the fluid is

$$Q_w = -k \left. \frac{\partial T}{\partial Y} \right|_{Y=0} \quad (41)$$

Written in dimensionless form, it becomes

$$q_w = - \left. \frac{\partial \theta}{\partial y'} \right|_{y'=0} \quad (42)$$

Transformed into (ξ, z) coordinate system, it is

$$q_w = -t^{-1/2} \gamma H \xi^{-1/2} \left(G - \frac{1}{\gamma} \right) [g'_0(0) + \xi g'_1(0) + \xi^2 g'_2(0)] \quad (43)$$

The values of g'_0 , g'_1 , and g'_2 are, respectively, 0.9575, 0.15074, and -0.25445. Figure 7 and Table 3 show that at a given instant the heat transfer increases rapidly behind the wavefront until it approaches a maximum, and then likely decreases. This can be explained as a result of an interplay among the expansion of the freestream, the growing boundary-layer thickness, and the viscous heating. From Eq. (2) we notice that the freestream temperature decreases with increasing distance from the wavefront. Consequently, heat transfer is from the wall to the fluid. On the other hand, the combined effect of the growing boundary-layer thickness and the viscous heating provides a resistance to the heat transfer further downstream. Equation (43) shows that the heat transfer at a given location decreases with respect to time and is approximately proportionate to $t^{-1/2}$. Physically this is reasonable because at a given location the boundary layer grows in time after the expansion wave passes by. The growth of the boundary layer deters the heat transfer.

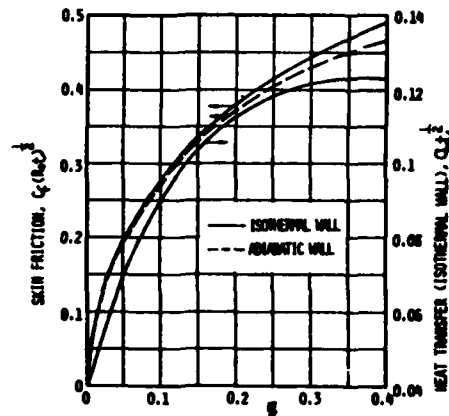


Figure 7. Skin Friction and Heat Transfer

TABLE 3. SKIN FRICTION AND HEAT TRANSFER

ξ	$C_f (Ret)^{1/2}$		$q_w (Ret)^{1/2}$
	Isothermal	Adiabatic	Isothermal
0	0.0000	0.0000	0.0000
0.025	0.1469	0.1465	0.0491
0.050	0.2054	0.2042	0.0675
0.075	0.2487	0.2465	0.0803
0.100	0.2839	0.2805	0.0901
0.125	0.3138	0.3091	0.0978
0.150	0.3398	0.3337	0.1041
0.175	0.3627	0.3553	0.1090
0.200	0.3833	0.3743	0.1130
0.225	0.4018	0.3912	0.1161
0.250	0.4185	0.4063	0.1186
0.275	0.4337	0.4199	0.1205
0.300	0.4476	0.4321	0.1218
0.325	0.4603	0.4431	0.1226
0.375	0.4824	0.4618	0.1231
0.400	0.4920	0.4697	0.1228
0.450	0.5087	0.4831	0.1214
0.500	0.5224	0.4939	0.1191

The coefficient of local heat transfer is determined by Newton's cooling law

$$Q_w = h(T_w - T_1)$$

or

$$h = q_w R (Re k)^{-1/2} [(1/\gamma) - G]^{-1} \quad (44)$$

where R is the gas constant. The function G is given in Eq. (2).

D. Skin Friction

The friction coefficient on the wall is defined as

$$C_f = \frac{1}{\rho U_r} \left. \frac{\partial U}{\partial Y} \right|_{Y=0} \quad (45)$$

After coordinate transformation and series expansion, Eq. (45) becomes

$$C_f = (Re t \xi)^{-1/2} FH[f_0''(0) + \xi f_1''(0) + \xi^2 f_2''(0)] \quad (46)$$

The values of f_0'' , $f_1''(0)$, and $f_2''(0)$ are, respectively, 1.1284, 0.7946, 0.41781 for the isothermal wall and 1.1284, 0.65896, 0.32903 for the adiabatic wall. The plot of $C_f(Re t)^{1/2}$ is also given in Figure 7 and Table 3. There is no large difference in skin friction between the isothermal wall and the adiabatic wall because both have very similar velocity profiles as shown in Figures 4 and 5. Specifically, at $\xi = 0.3$ the skin friction for the isothermal wall is approximately 5.5 percent higher than that for the adiabatic wall.

V. SUMMARY AND CONCLUSIONS

Solutions were obtained for the unsteady compressible laminar boundary-layer flow that develops within a centered expansion wave for isothermal and adiabatic walls. The solutions were obtained by a method of similarity transformation via one-parameter groups and a power series expansion in terms of a dimensionless distance ξ from the expansion wavefront. The series expansion includes terms up to second order of ξ , and thus constitutes an improvement over Hall's zero-order solution for the temperature in the boundary layer over an adiabatic wall. The present results show that at a given location the isothermal wall surface temperature is higher than the adiabatic wall surface temperature by a factor of approximately 0.335 ξ .

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APPENDIX A

Function F^* in Eq. (33) and Function G^* in Eq. (34)

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Function F^* of Eq. (33)

$$\begin{aligned}
 F^*(z) = & \left(\frac{3\gamma-1}{\gamma+1}\right) f_1''' - \left(\frac{1}{2} \frac{\gamma-1}{\gamma+1}\right) z f_1'' - \left(\frac{3}{\gamma+1}\right) f_0 f_1'' \\
 & - \left(\frac{5}{2} + 2 \frac{\gamma-1}{\gamma+1}\right) f_1' + \left(\frac{6}{\gamma+1}\right) f_0' f_1' - \left(\frac{5}{\gamma+1}\right) f_0'' f_1 \\
 & - \frac{3\gamma-1}{(\gamma+1)^2} f_0''' + 3 \left(\frac{\gamma-1}{(\gamma+1)^2}\right) f_0'' f_0 + \frac{3}{2} \frac{\gamma-1}{\gamma+1} f_0' \\
 & - 2 \frac{\gamma-1}{(\gamma+1)^2} f_0' f_0' + 2 \frac{\gamma-1}{\gamma+1} g_1 - \left(\frac{\gamma-1}{\gamma+1}\right)^2 g_2
 \end{aligned}$$

Function G^* of Eq. (34)

$$\begin{aligned}
 G^*(z) = & \frac{1}{Pr} \left(\frac{1}{2} \frac{\gamma-1}{\gamma+1} + \frac{2\gamma}{\gamma+1}\right) g_1'' + \left(\frac{1}{4} \frac{\gamma-1}{\gamma+1} z\right) g_1' \\
 & - \frac{3}{\gamma+1} f_0 g_1' - \left(2 - \frac{\gamma-1}{2(\gamma+1)}\right) g_1 + \frac{4}{\gamma+1} f_0' g_1 \\
 & - \frac{1}{Pr} \left(\frac{\gamma(\gamma-1)}{(\gamma+1)^2} + \frac{\gamma}{\gamma+1}\right) g'' - \frac{5}{\gamma+1} f_1 g_0' \\
 & + \left(\frac{3}{2} \frac{\gamma-1}{(\gamma+1)^2}\right) f_0 g_0' - \left(\frac{\gamma-1}{\gamma+1} - \left(\frac{\gamma-1}{\gamma+1}\right)^2\right) g_0 \\
 & - \frac{4\gamma}{(\gamma+1)^2} f_0'^2 - \frac{2(\gamma-1)}{(\gamma+1)^2} f_0' - \frac{2}{\gamma+1} f_1' + \left(2 \frac{\gamma-1}{(\gamma+1)^2}\right) f_0' g_0 \\
 & + \frac{2}{\gamma+1} f_1' g_0 + \frac{\gamma-1}{\gamma+1} - \left(\frac{\gamma-1}{\gamma+1}\right)^2 - \frac{4}{\gamma+1} f_0'' f_1''
 \end{aligned}$$

NOMENCLATURE

C_f	= coefficient of local skin friction
C_p	= specific heat at constant pressure
F	= inviscid velocity function, Eq. (1)
F^*	= right-hand side of Eq.(33), Appendix A
f	= transformed streamfunction, Eq. (21)
f_n	= transformed streamfunction, Eq. (26), $n = 1,2,3,\dots$
G	= inviscid temperature function, Eq. (2)
G^*	= right-hand side of Eq. (34), Appendix A
g	= transformed temperature function, Eq. (22)
g_n	= transformed temperature function, Eq. (28)
H	= inviscid pressure function, Eq. (3)
h	= heat transfer coefficient
k	= thermal conductivity
L	= reference length
n	= integer, $n = 0,1,2,\dots$
p	= dimensionless pressure
\bar{p}	= dimensional pressure
P_1	= dimensionless freestream pressure
Pr	= Prandtl number
Re	= Reynolds number
Q_w	= dimensional heat flux at the wall
q_w	= dimensionless heat flux at the wall
R	= gas constant

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T = dimensional temperature
 T_1 = dimensional freestream temperature
 T_0 = dimensional temperature of the undisturbed fluid
 T_r = reference temperature $U_r^2/R = \gamma T_0$
 T_w = dimensional wall temperature
 t = dimensionless time $\bar{t}U_r/L$
 \bar{t} = dimensional time
 t^* = dimensionless time, Figure 1
 U = x-component velocity, dimensional
 U_r = reference velocity, speed of the expansion wavefront $\sqrt{\gamma RT_0}$
 u = x-component velocity, dimensionless, Eq. (6)
 u_1 = dimensionless freestream velocity
 V = y-component velocity, dimensional
 v = y-component velocity, dimensionless, Eq. (6)
 x, y' = dimensionless coordinates, Figure 1 and Eq. (6)
 \bar{x}, Y = dimensional coordinates
 y = dimensionless, transformed coordinate y' , Eq. (11)
 z = variable, Eq. (25)
 γ = ratio of specific heats
 η = similarity variable, Eq. (20)
 θ = dimensionless temperature
 θ_1 = dimensionless freestream temperature
 θ_w = dimensionless wall temperature
 μ = dimensionless viscosity

$\bar{\mu}$ = dimensional viscosity
 μ_r = reference viscosity
 ξ = similarity variable, Eq. (4)
 ρ = dimensionless density, Eq. (6)
 $\bar{\rho}$ = dimensional density
 ρ_r = reference density
 ψ = streamfunction, Eq. (12)
 ∞ = outer edge of the boundary layer

Subscripts

i = inviscid
 r = reference
 w = wall condition
 0 = undisturbed

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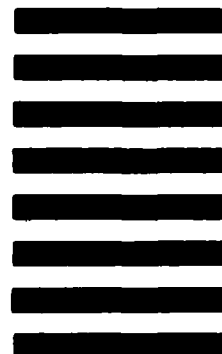


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